Probability (2)

Probability

Definition

- ☺ Is the chance of something happening.
- ☺ We used fractions, decimals or percent to find probability.

Example #1

➢ *Tossing a coin*

The sample space is $\Omega = \{H;T\}$

Each outcome has 50% or 0.5 or $\frac{1}{2}$ chance to appear.

We said that $P(H) = \frac{1}{2} = 0.5 = 50\%$

Probability

Definition

- ☺ Is the chance of something happening.
- ☺ We used fractions, decimals or percent to find probability.

Example #2

➢ *Throwing a fair dice*

The sample space is $\Omega = \{1;2;3;4;5;6\}$

The dice is fair

Equal chance for each of the outcomes to appear: $P = \frac{1}{6}$

Probability

Definition

- ☺ Is the chance of something happening.
- ☺ We used fractions, decimals or percent to find probability.

Example #3

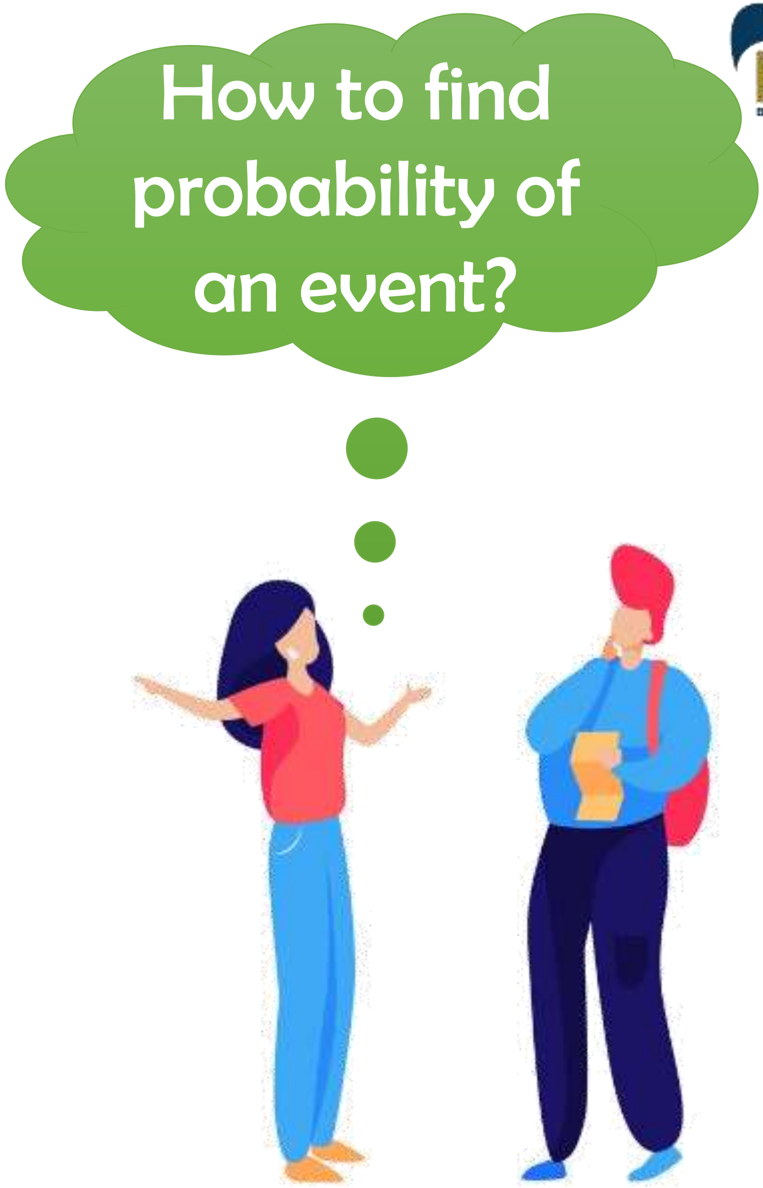
➢ ***Throwing an unfair dice***

The sample space is $\Omega = \{1;2;3;4;5;6\}$

$$P(1) = P(2) = \frac{1}{5} = 0.2 = 20\%$$

$$P(3) = P(4) = \frac{1}{4} = 0.25 = 25\%$$

$$P(5) = P(6) = \frac{1}{20} = 0.05 = 5\%$$



Probability

1 Equiprobability

2 Non Equiprobability

$$P(A) = \frac{\text{card}(A)}{\text{card}(\Omega)} = \frac{\text{nb of favorable outcomes}}{\text{nb of possible outcomes}}$$

Throwing a fair dice:

Ω = {1;2;3;4;5;6}

A: “the obtained number is less than 5}

A = {1;2;3;4}

$$P(A) = \frac{card(A)}{card(\Omega)} = \frac{4}{6} = \frac{2}{3}$$

Probability

1 Equiprobability

2 Non Equiprobability

A is an event.
If A = {w1;w2;…}
P(A) = P({w1})+P({w2})+…

Throwing a dice :

- 2 faces are numbered 1
- 3 faces are numbered 2
- 1 face is numbered 3

The sample space is $\Omega = \{1;2;3\}$

A: “the obtained number is odd”
A = {1;3}

P(A) = P({1}) + P({3})

$$= \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

Probability

Properties

- $P(\Omega) = 1$
- $P(\phi) = 0$
- $A \subseteq \Omega$, $0 \leq P(A) \leq 1$
- $P(A) + P(\bar{A}) = 1$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If A and B are mutually exclusive events, then $P(A \cap B) = 0$
- $A \subseteq B$; $P(A \cap B) = P(A)$ and $P(A \cup B) = P(B)$
- A is the opposite event of B so:
  - $A \cap B = \phi$
  - $P(A) + P(B) = 1$

Probability

Application # 1

A coin is tossed 3 times.

1. What is the number of all possible outcomes?

2	2	2

$= 2 \times 2 \times 2 = 8$ outcomes

Probability

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1. What is the number of all possible outcomes?

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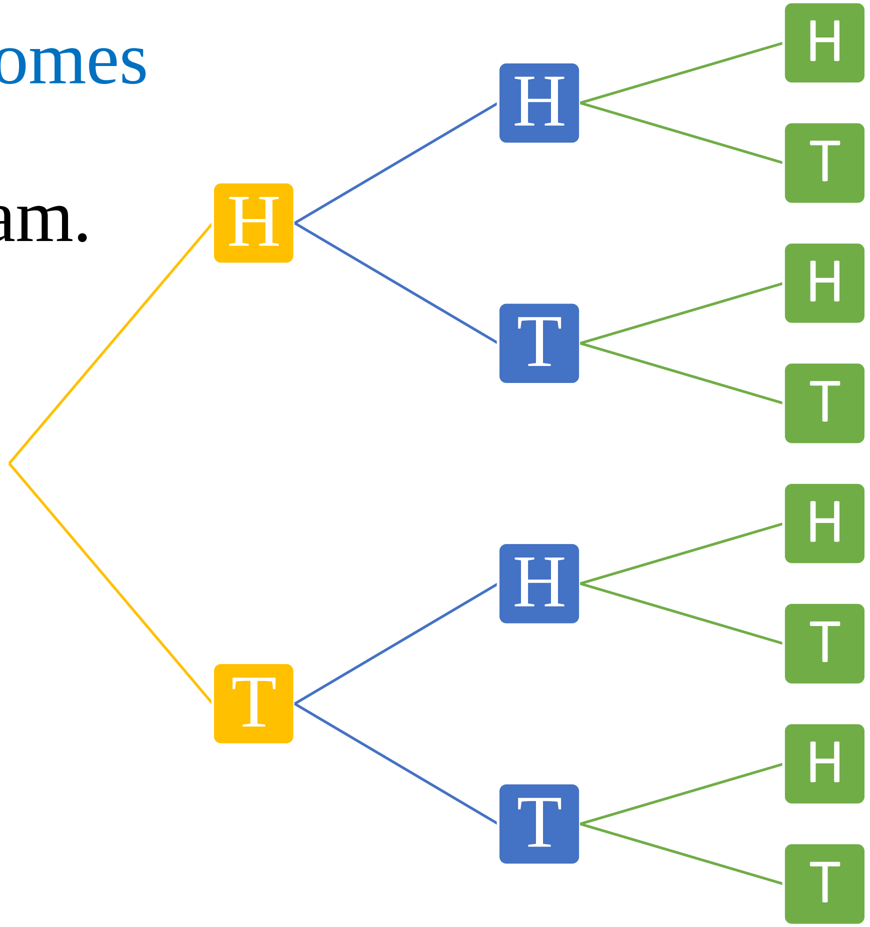
2. List the sample space using tree diagram.



$\Omega = \{HHH ; HHT ; HTH ; HTT; THH ; THT ; TTH ; TTT\}$

Probability

Application # 1

Ω = {HHH ; HHT ; HTH ; HTT; THH ; THT ; TTH ; TTT}

A coin is tossed 3 times.

3. Calculate the probability of the following events:

A: “the three outcomes are tail” A= {TTT}

$P(A) = \frac{1}{8}$

B: “the first two outcomes are head” B= {HHH ; HHT}

$P(B) = \frac{2}{8} = \frac{1}{4}$

C: “exactly two among the three outcomes are tail” C= {HTT;THT;TTH}

$P(C) = \frac{3}{8}$

D: “at least two outcomes are head” D= {HHH;HHT;HTH;THH}

$P(D) = \frac{4}{8} = \frac{1}{2}$

Probability

Application # 2

A balanced cubic die is thrown once, the faces of which are numbered from 1 to 6. The faces 1, 2 and 3 are colored red, the face 4 is colored blue and the faces 5 and 6 are colored green.

Part A: We note the number appearing on the upper face.

1) Is it an equiprobable situation? Justify.

   Yes, since the die is balanced and each number exists one time.

2) Write the sample space Ω.

   $\Omega = \{ 1 ; 2 ; 3 ; 4 ; 5 ; 6\}$

3) Calculate the probability of each number.

   $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$

Probability

Application # 2

A balanced cubic die is thrown once, the faces of which are numbered from 1 to 6. The faces 1, 2 and 3 are colored red, the face 4 is colored blue and the faces 5 and 6 are colored green.

Part B: We note the color of the upper face.

1) Is it an equiprobable situation? Justify.

No, since the color red exists on 3 faces while the color blue exists on 1 face.

2) Write the sample space Ω.

$\Omega = \{Red\ ;\ Green\ ;\ Blue\}$

3) Calculate the probability of each number.

$P(Red) = \frac{3}{6} = \frac{1}{2}$ (3 faces out of 6 faces)

$P(Blue) = \frac{1}{6}$ (one face out of 6 faces)

$P(Green) = \frac{2}{6} = \frac{1}{3}$ (2 faces out of 6 faces)

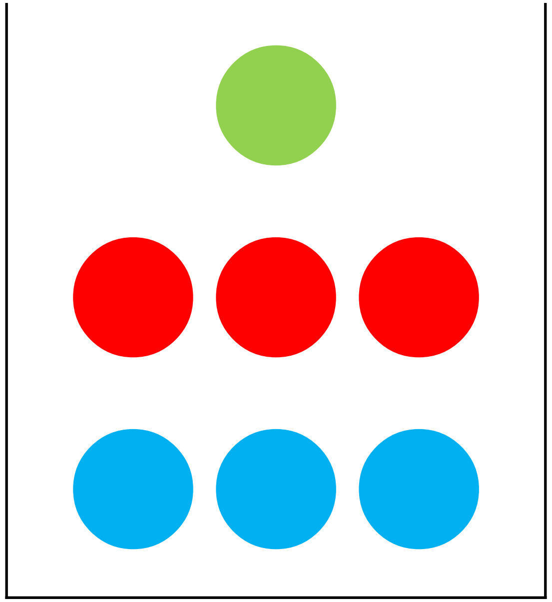
$P(Red) + P(Green) + P(Blue) = 1$

Probability

Application # 3

An urn contains 7 balls:

- 3 red balls.
- 3 blue balls.
- 1 green ball.



Part A:

1 ball is selected randomly and successively from the urn.

Calculate the probability of the following events:

A: “the ball is red” $P(A) = \frac{3}{7}$

B: “the ball is blue” $P(B) = \frac{3}{7}$

C: “the ball is green” $P(A) = \frac{1}{7}$

Probability

Application # 3

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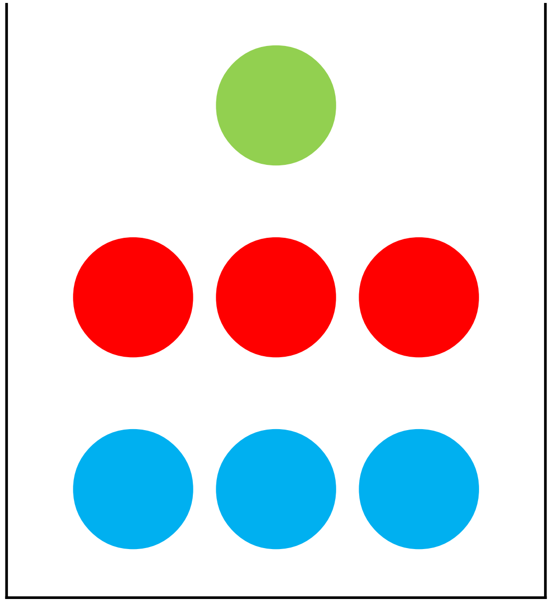

Part B:

2 balls are selected randomly and **successively without replacement** from the urn.

Calculate the probability of the following events:

A: “the balls are red”

$$P(A) = \frac{A_3^2}{A_7^2} = \frac{1}{7}$$

Second method:

$$Card(\Omega) = 7 \times 6 = 42$$

$$Card(A) = 3 \times 2 = 6$$

$$P(A) = \frac{6}{42} = \frac{1}{7}$$

Probability

Application # 3

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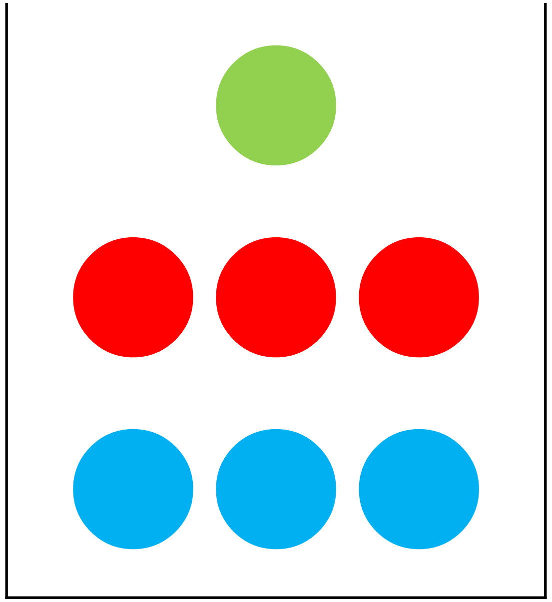


<u>Part B:</u>

2 balls are selected randomly and **successively without replacement** from the urn.

Calculate the probability of the following events:

B: “the balls are of same color”

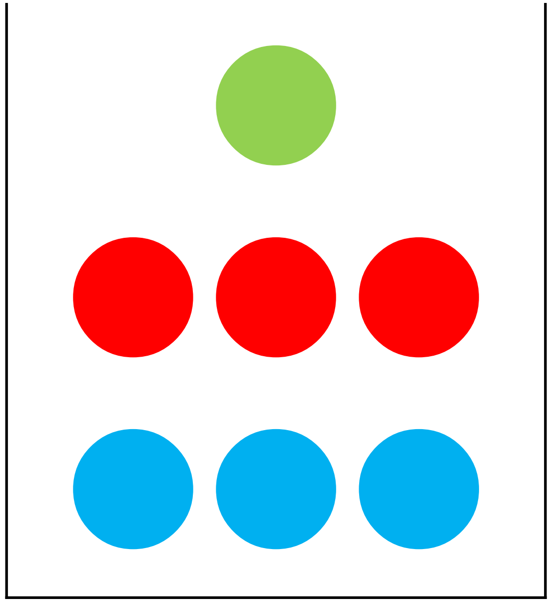
P(B) = P(RR) + P(BB) = $\frac{1}{7} + \frac{A_3^2}{A_7^2} = \frac{2}{7}$

Probability

Application # 3

An urn contains 7 balls:

- 3 red balls.
- 3 blue balls.
- 1 green ball.



Part B:

2 balls are selected randomly and **successively without replacement** from the urn.

Calculate the probability of the following events:

C: “only the first ball is red”

$$P(C) = P(R\overline{R}) = \frac{A_3^1 \times A_4^1}{A_7^2} = \frac{12}{42} = \frac{2}{7}$$

D: “the first ball is red”

$$P(D) = P(R\overline{R}) + P(RR) = \frac{2}{7} + \frac{1}{7} = \frac{3}{7}$$

Probability

Application # 3

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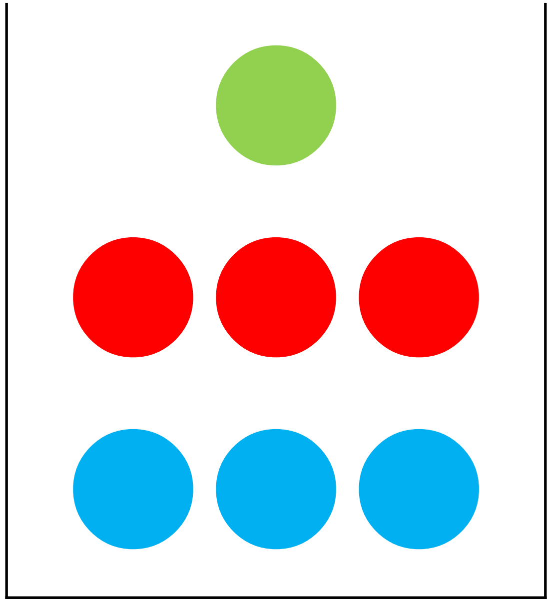


<u>Part B:</u>

2 balls are selected randomly and **successively without replacement** from the urn.

Calculate the probability of the following events:

E: “only one ball is red”

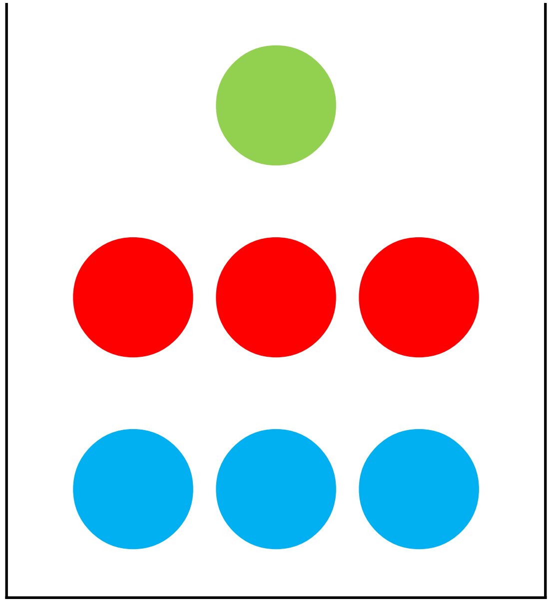
$P(E) = P(R\overline{R}) + P(\overline{R}R) = P(C)\times 2 = \frac{4}{7}$

Probability

Application # 3

An urn contains 7 balls:

- 3 red balls.
- 3 blue balls.
- 1 green ball.



Part C:

3 balls are selected randomly and **successively with replacement** from the urn.

Calculate the probability of the following events:

A: “the balls are of same color”

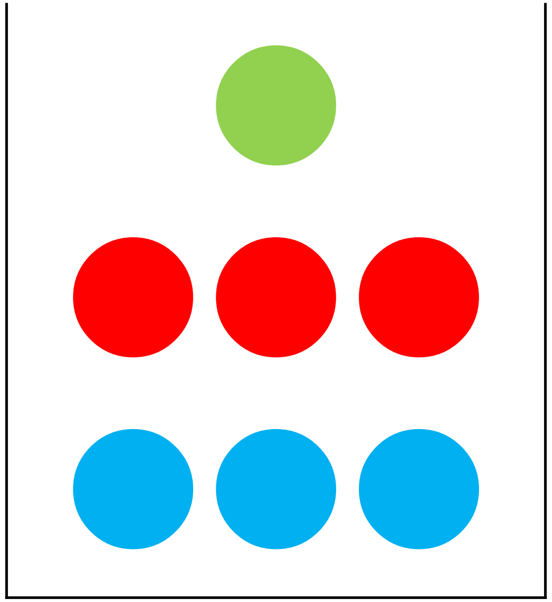
$$P(A) = P(RRR) + P(BBB) + P(GGG) = \left(\frac{3}{7}\right)^3 + \left(\frac{3}{7}\right)^3 + \left(\frac{1}{7}\right)^3 = \frac{55}{343}$$

Probability

Application # 3

An urn contains 7 balls:

- 3 red balls.
- 3 blue balls.
- 1 green ball.



<u>Part C:</u>

3 balls are selected randomly and **successively with replacement** from the urn.

Calculate the probability of the following events:

B: “one ball is blue”

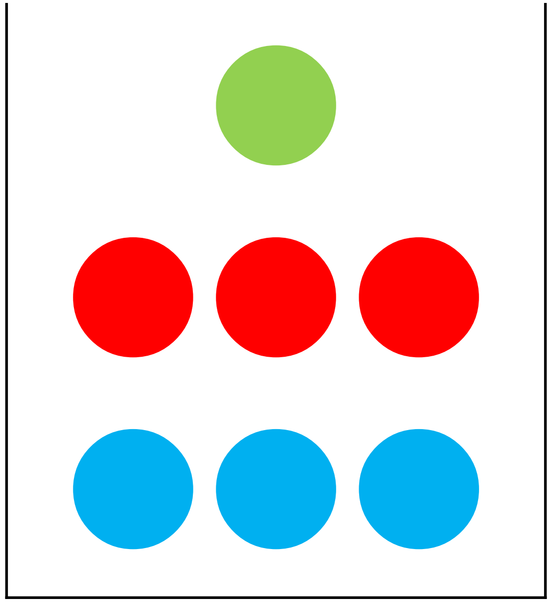
$$P(B) = P(B\overline{B}\overline{B}) \times \frac{3!}{2! \times 1!} = \frac{1}{7} \times \left(\frac{4}{7}\right)^2 \times 3 = \frac{48}{343}$$

Probability

Application # 3

An urn contains 7 balls:

- 3 red balls.
- 3 blue balls.
- 1 green ball.



Part C:

3 balls are selected randomly and **successively with replacement** from the urn.

Calculate the probability of the following events:

B: “at least one ball is blue”

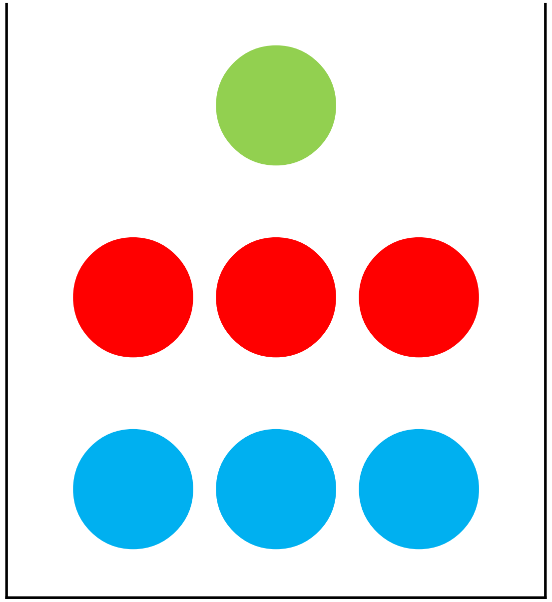
P(B) = 1- P(“no ball is blue”) = 1- $\left(\frac{4}{7}\right)^3 = \frac{279}{343}$

Probability

Application # 3

An urn contains 7 balls:

- 3 red balls.
- 3 blue balls.
- 1 green ball.



Part C:

3 balls are selected randomly and **successively with replacement** from the urn.

Calculate the probability of the following events:

C: “the three balls are of three different colors”

$P(C) = P(RBG) \times 3! = \frac{3}{7} \times \frac{3}{7} \times \frac{1}{7} = \frac{9}{343}$

Probability

Application # 3

An urn contains 7 balls:

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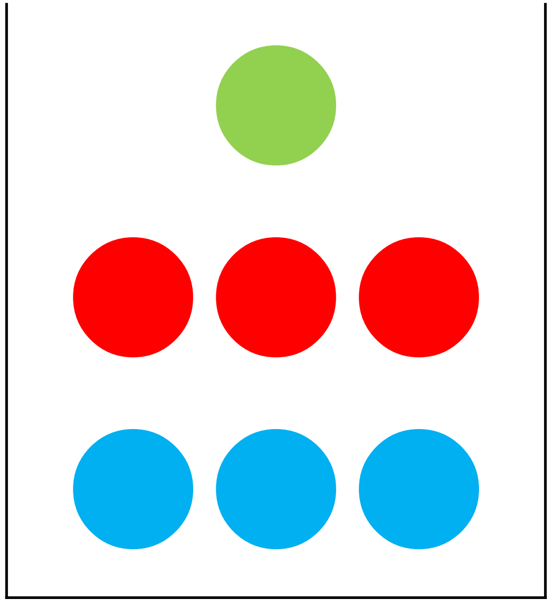
- 1 green ball.



<u>Part C:</u>

3 balls are selected randomly and **successively with replacement** from the urn.

Calculate the probability of the following events:

D: “the balls are of different colors”

P(D) = 1- P(‘same color”) = 1 – P(A) = $\frac{288}{343}$

Probability

Application # 4

The adjoining table shows the distribution of 72 students of the three sections of the second secondary classes in a school.
One student is chosen randomly.
Calculate the probability of the following events:
A: “the student is from section A”

Class	Section A	Section B	Section C	Total
Boys	12	16	11	39
Girls	15	10	8	33
Total	27	26	19	72

$$P(A) = \frac{27}{72} = \frac{3}{8}$$

Probability

Application # 4

The adjoining table shows the distribution of 72 students of the three sections of the second secondary classes in a school.
One student is chosen randomly.
Calculate the probability of the following events:
B: “the student is a girl”

Class	Section A	Section B	Section C	Total
Boys	12	16	11	39
Girls	15	10	8	33
Total	27	26	19	72

$$P(B) = \frac{33}{72} = \frac{11}{24}$$

Probability

Application # 4

The adjoining table shows the distribution of 72 students of the three sections of the second secondary classes in a school.
One student is chosen randomly.
Calculate the probability of the following events:
C: “the student is a girl from section A”

Class	Section A	Section B	Section C	Total
Boys	12	16	11	39
Girls	15	10	8	33
Total	27	26	19	72

$$P(C) = \frac{15}{72} = \frac{5}{24}$$

Probability

Application # 4

The adjoining table shows the distribution of 72 students of the three sections of the second secondary classes in a school.
One student is chosen randomly.
Calculate the probability of the following events:
D: “the student is a girl or from section A”

Class	Section A	Section B	Section C	Total
Boys	12	16	11	39
Girls	15	10	8	33
Total	27	26	19	72

$$P(D) = P(B) + P(A) - P(B \cap A) = \frac{11}{24} + \frac{3}{8} - \frac{5}{24} = \frac{15}{24} = \frac{5}{8}$$

Probability

Application # 4

The adjoining table shows the distribution of 72 students of the three sections of the second secondary classes in a school.
One student is chosen randomly.
Calculate the probability of the following events:
E: "the students is a boy and not from section A"

Class	Section A	Section B	Section C	Total
Boys	12	16	11	39
Girls	15	10	8	33
Total	27	26	19	72

$P(E) = \frac{27}{72} = \frac{3}{8}$ (the boy is from section B or section C i.e. 16+11=27 out of 72)